Performance Analysis of And-Or Radar CFAR Detectors

Sher Bahadur Yadav
Electronics and Radar Development Establishment (LRDE) Bangalore, INDIA
sherbahadur.yadav@lrde.drdo.in

Abstract—Radar detection procedures involve the comparison of the received signal amplitude to a threshold. In order to obtain a constant false-alarm rate (CFAR), an adaptive threshold must be applied reflecting the local clutter situation. Their primary goal is to maintain the desired false alarm rate and to be invariant to changes in the clutter density function. This is achieved by adaptively estimating the clutter power based on a finite number of clutter samples within a processing window. To achieve this, a CFAR detector processes a finite set of range-Doppler samples within a reference window surrounding the cell under test and sets the threshold adaptively based on a local estimate of the total noise power.

The Cell Averaging (CA) CFAR detector is optimal for detecting targets embedded in exponential clutter and noise of unknown power, utilizing maximum likelihood estimate of the noise power to set the adaptive threshold. The Ordered Statistics (OS) CFAR detector is robust in rejecting impulsive noise and preserving edges. The And-Or CFAR detector combine the result of the CA-CFAR and OS-CFAR to get a better detection performance. The paper presents experimental simulations carried out for And-Or CFAR. The performance of them has been evaluated and compared against CA, OS and Trimmed Mean (TM) CFAR.

Keywords: CFAR detector, CA, OS, TM

I. INTRODUCTION

The received signal in a radar system is accompanied by thermal noise and clutter. Since the environment in which a radar operates depends on factors such as weather conditions and the physical location of operation, the returned signals are statistically non stationary, with unknown variance at the receiver input. Thus, the ideal detector using a fixed threshold is extremely sensitive to the total noise (thermal noise plus clutter) variance. In fact, a small increase in the total noise power results in a corresponding increase of several orders of magnitude in the probability of false alarm. To overcome this problem, CFAR detectors are commonly used. CFAR automatically raises the threshold level to keep clutter echoes and external noise from overloading the automatic tracker with extraneous information. Various CFAR design schemes have been developed. Finn and Johnson [1] first presented the CA-CFAR method. The algorithm is relatively simple in that it computes average of the signal within a sliding window (cell) as the threshold for detection. This adaptive method can play an effective part in noise and clutter environments, and provide nearly the best ability of signal detection while preserving the constant false alarm rate. Rohling [2] proposed the OS-CFAR detector. This detector possesses the ability to counter multiple targets. The OS-CFAR rank orders the samples in the CFAR reference window and selects one sample as the CFAR statistic. The CFAR is thus capable of rejecting interfering targets. In addition, an OS-CFAR is capable of suppressing clutter edge false alarms provided the order of selected sample is greater than half of window size. With the Greatest-Of CA (GOCA) CFAR detector [3]–[5], the problem of increase of the false alarm probability due to the presence of a step discontinuit in the distributed clutter cloud has been treated. Trunk [6] proposed the Smallest-Of CA (SOCA) CFAR detector to improve the resolution of closely spaced targets. The SOCA-CFAR estimates the interference power in the lagging and leading reference windows and selects the smaller of the two estimates as the CFAR statistic. Rickard and Dillard [7] and Ritcey [8] define a censored CFAR, which rank orders the measured samples in the reference window and discards the largest few samples prior to computing the CFAR statistic. A modified OS-CFAR detector, known as the ”trimmed mean” (TM) CFAR detector which implements trimmed averaging after ordering, is also considered. By judiciously trimming the ordered samples, the TM-CFAR detector may actually perform somewhat better than the OS-CFAR detector. A second-order statistic called the variability index (VI) and the ratio of the means of the leading and lagging windows were calculated to dynamically adjust the background estimation. In [9], two versions of the CFAR detectors, i.e. And-CFAR and Or-CFAR, were proposed by making use of the two threshold settings from CA-CFAR and OS-CFAR. The detection is based on whether both (And-CFAR) or either (Or-CFAR) threshold criteria are met. In spite of the availability of numerous versions of CFAR algorithm, they are often tailored to specific types of frequency spectrums and application.

Figure 1 contains a plot of the CA, OS and TM-CFAR thresholds derived from the simulated returns. The detectors are designed to achieve a *probability of false alarm* 10^{-6}. The reference window consists of 16 cells. The TM-CFAR discards the largest and smallest two, two samples respectively, and the OS-CFAR uses the 12th sample to compute the CFAR statistic. In this example, both the TM and OS-CFARs detect the four targets, whereas the CA-CFAR detects only the three target.

As shown in Figure 2, And-Or CFAR detector consists of the threshold calculation modules for CA-CFAR and OSCFAR schemes. The obtained threshold values are multiplied by the scaling factor α and compared against the signal under detection Y.

In this paper the performance of And-Or CFAR detector is investigated and same is compared with CA, OS and TM-CFAR detectors.

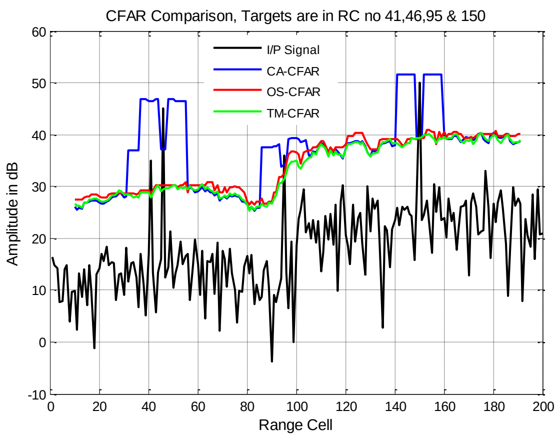

Figure 1: OS and TM CFAR mitigate mutual target masking while CA-CFAR misses one target, N = 16, *k* = 12.

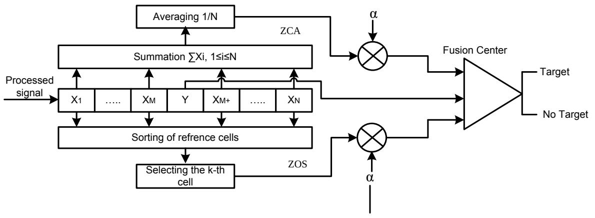

Figure 2: Architecture of AND-OR CFAR Detector

II. AND-OR CFAR DETECTOR

Factors that affect the false alarm rate in signal detection include 1) the radar environment 2) the distribution of the reflection noise, and 3) the selection of threshold value. Detection decisions can be applied to signals present at various stages of the radar signal processing, from raw echoes to heavily preprocessed data such as range data, Doppler spectra or even synthetic aperture radar images. In our case each range bin for each pulse can be individually tested to decide if a target is present at the range corresponding to the range bin, and the spatial angles corresponding to the antenna pointing direction for that pulse. We also assume that probability density function (PDF) of each cell X_i within the reference window as [10].

$$p_{Y_0}(x) = p_{X_i}(\mathrm{x}) = \frac{1}{\mu}\exp\left(\frac{-x}{\mu}\right), x \geq 0 \qquad (1)$$

The cumulative distribution function (CDF) of X_i then becomes

$$P_{Y_0}(x) = P_{X_i}(\mathrm{x}) = 1 - \exp\left(\frac{-x}{\mu}\right), x \geq 0 \qquad (2)$$

The And-Or CFAR detection is a combination of CA and OS CFAR. In CA-CFAR detector, the threshold value (Z_{CA}) is calculated as the average of all the cells within the reference window centered at the cell under test.

$$Z_{CA} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (3)$$

Where N is the size of the reference window. Since each component X_i has a PDF as indicated in (1), the PDF of the threshold value exhibits a Gamma distribution.

$$f_{CA}(z) = \frac{1}{\mu\Gamma(N)}\left(\frac{z}{\mu}\right)^{(N-1)}\exp\left(-\frac{z}{\mu}\right) \qquad (4)$$

The corresponding CDF shows a Erlang distribution

$$F_{CA}(z) = 1 - \exp\left(-\frac{z}{\mu}\right)\sum_{i=0}^{N-1}\frac{(\frac{z}{\mu})^i}{i!} \qquad (5)$$

In the OS-CFAR detector, the threshold value Z_{OS} is defined as the k^{th} largest value in the reference window. It's PDF and CDF can be expressed as follows

$$f_{os}(z) = k\binom{N}{k}[1 - P_X(z)]^{k-1}p_X(z)$$

$$= \frac{k}{\mu}\binom{N}{k}\exp\left(\frac{z}{\mu}\right)^{(N-k+1)}\left(1 - \exp\left(-\frac{z}{\mu}\right)\right)^{(k-1)} \qquad (6)$$

$$F_{os}(z) = \sum_{i=k}^{N}\binom{N}{i}\left(1 - \exp\left(-\frac{z}{\mu}\right)\right)^k\exp\left(-\left(\frac{z}{\mu}\right)^{N-k}\right) \qquad (7)$$

A. *And-CFAR*

For And-CFAR, a target signal Y is detected when its value is greater than the maximum of the two threshold values times a scaling factor α.

The decision criterion for this algorithm is

$$Z_{and} = \max(Z_{CA}.\propto, Z_{OS}.\propto);$$

$$Y \geq Z_{and} : \mathcal{H}_1; \quad Y < Z_{and} : \mathcal{H}_0 \qquad (8)$$

Where Z_{and} is the And-CFAR adaptive threshold, Z_{CA} and Z_{OS} are the CA-CFAR and OS-CFAR estimated noise levels. In which $\mathcal{H}_1$ epresents target present and $\mathcal{H}_0$ represents no target. The PDF of Z_{and} can be calculated as [11]

$$f_{And}(z) = f_{CA}(z)F_{OS}(z) + f_{OS}(z)F_{CA}(z) \qquad (9)$$

In homogeneous background, with all the cells are IID, the false alarm probability $P_{FA} = P(\mathcal{H}_1|\mathcal{H}_0)$ and the detection probability $P_D = P(\mathcal{H}_1|\mathcal{H}_1)$ of the And-CFAR scheme are:

$$P_{FA} = k\binom{N}{k}\left\{\frac{\Gamma(N-k+\alpha+1)\Gamma(k)}{\Gamma(N+\alpha+1)} - \frac{1}{N}\sum_{i=0}^{N-1}\sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j\left(\frac{N}{2N-k+j+\alpha+1}\right)^{i+1}\right\} + \sum_{i=k}^{N}\binom{N}{i}\sum_{j=0}^{j}\binom{i}{j}(-1)^j\left(\frac{N}{2N+j-i+\alpha}\right)^N \qquad (10)$$

$$P_D = k\binom{N}{k}\left\{\frac{\Gamma\left(N-k+\frac{\alpha}{(1+\chi)}+1\right)\Gamma(k)}{\Gamma\left(N+\frac{\alpha}{(1+\chi)}+1\right)} - \frac{1}{N}\sum_{i=0}^{N-1}\sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j\left(\frac{N}{2N-k+j+\frac{\alpha}{(1+\chi)}+1}\right)^{i+1}\right\} + \sum_{i=k}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j\left(\frac{N}{2N+j-i+\alpha/(1+\chi)}\right)^N \qquad (11)$$

where χ is signal to noise ratio.

B. *Or-CFAR*

For Or-CFAR, a target signal Y is detected when its value is greater than any the CA-CFAR threshold and the OS-CFAR threshold, which is equivalent as choosing the minimum value of the threshold.
The decision criterion for this algorithm is

$$Z_{or} = \min(Z_{CA}.\alpha, Z_{OS}.\alpha) \ ;$$

$$\begin{array}{llll} Y & \geq & Z_{and} & : \mathcal{H}_1; \\ Y & < & Z_{and} & : \mathcal{H}_0 \end{array} \qquad (12)$$

In which $\mathcal{H}_1$ represents target present and $\mathcal{H}_0$ represents no target. The PDF of Z_{or} can be calculated as [11]

$$\begin{aligned} F_{or}(z) &= f_{CA}(z)[1 - F_{OS}(z)] + f_{OS} \\ &= f_{CA}(z) + f_{OS}(z) \\ &-[f_{CA}(z)F_{OS}(z) + f_{OS}(z)F_{CA}(z)] \end{aligned} \qquad (13)$$

In homogeneous background, with all the cells are IID, the false alarm probability $P_{FA}= P(\mathcal{H}_1|\mathcal{H}_0)$ and the detection probability $P_D= P(\mathcal{H}_1|\mathcal{H}_1)$ of the Or-CFAR scheme are:

$$\begin{aligned} P_{FA} &= \frac{1}{(1+\alpha/N)^N} + k\binom{N}{k} * \\ &\frac{1}{N}\sum_{i=0}^{N-1}\sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j\left(\frac{N}{2N-k+j+\alpha+1}\right)^{i+1} - \\ &\sum_{i=k}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j\left(\frac{N}{2N+j-i+\alpha}\right)^N \end{aligned} \qquad (14)$$

$$\begin{aligned} P_D &= \frac{1}{[1+\alpha/(1+\chi)/N]^N} + \frac{k}{N}\binom{N}{k} * \\ &\sum_{i=0}^{N-1}\sum_{j=0}^{k-1}\binom{k-1}{j}(-1)^j\left(\frac{N}{2N-k+j+\frac{\alpha}{(1+\chi)}} + 1\right)^{i+1} - \\ &\sum_{i=k}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j\left(\frac{N}{2N+j-i+\alpha/(1+\chi)}\right)^N \end{aligned} \qquad (15)$$

Note that P_{FA} does not depend on the actual interference power, but only on the number N of the neighboring cells averaged, order k and α the scaling factor.

III. SIMULATION RESULTS

The probability of detection achieved by And-Or CFAR detector has been evaluated by means of computer simulation and the same is compared by CA, OS and TM-CFAR detector.

In Figure 3, the detection probability with different P_{FA} and k value has been plotted. It can be seen that as k reaches 15, the P_D is minimum (For a fixed value of P_{FA}). This can be explained that as the selected reference cell in the OS processing is with large value, and the OS processed threshold has a higher value than CA processed threshold value, so the k value dominates the changes.

In case of Or-CFAR, the threshold from the OS-CFAR does not dominate all the time, which leads to the P_D performance as shown in Figure 4.

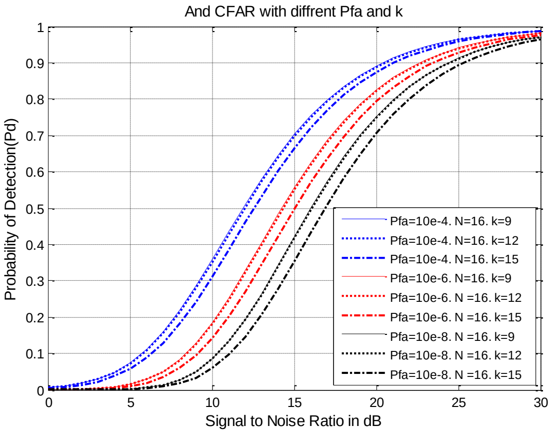

Figure 3: Simulations for different parameter setting in AND-CFAR

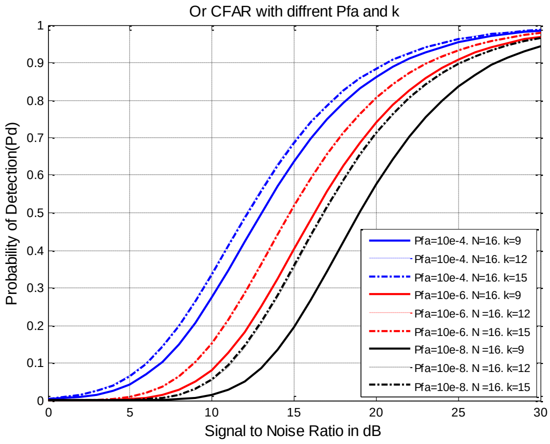

Figure 4: Simulations for different parameter setting in Or-CFAR

We may observe that the performance of detection probability and the false alarm probability are against each other. A stringent false alarm rate setting also degrades the detection rate. It is also observed that a smaller k value enhances the detection probability. To meet our system specification: $P_{FA} \leq 10^{-6}, P_D \geq 0.82, N = 16, k = 12,$ while SNR is not less than 20dB in And-CFAR, a set of simulation were conducted to determine the design parameters and results are summarized in Table 1 and 2 for AND and Or-CFAR respectively. The three value in each table entry are the detection probability corresponding to 10^{-4}, 10^{-6} and 10^{-8} probability of false alarm setting, respectively.

Figure 5 shows the probability of detection results of the five CFAR detectors, i.e. And, Or, TM, CA and OS subject to a constant $P_{FA} = 10^{-8}$. The value of N and k are 16 and 14 respectively. The no. of cells censored in TM-CFAR are two, one from each side. It is clear that And-CFAR scheme achieves the best detection probability among five. Or-CFAR better than TM and CA-CFAR but with a very close performance, CA and TM is better with the OS-CFAR.

Table I And-CFAR Simulations for (N, k, P_{FA}) Combinations

	$N = 8$	$N = 12$	$N = 16$	$N = 20$	$N = 24$

k	4 6 7	7 9 11	9 12 15	11 15 18	16 18 22
10dB	0.2862 0.1186 0.0353	0.3379 0.1591 0.0517	0.3597 0.1819 0.0595	0.3733 0.1962 0.0753	0.3840 0.2041 0.0141
15dB	0.6442 0.4624 0.2785	0.6876 0.5225 0.3316	0.7042 0.5515 0.3553	0.7141 0.5683 0.3959	0.7217 0.5775 0.2091
20dB	0.8657 0.7739 0.6459	0.8852 0.8078 0.6894	0.8923 0.8230 0.7077	0.8965 0.8316 0.7361	0.8997 0.8361 0.5926
25dB	0.9549 0.9209 0.8675	0.9618 0.9339 0.8867	0.9643 0.9396 0.8946	0.9658 0.9428 0.9063	0.9669 0.9445 0.8448
30dB	0.9855 0.9741 0.9557	0.9877 0.9785 0.9624	0.9885 0.9804 0.9652	0.9890 0.9815 0.9692	0.9894 0.9820 0.9478

Table II Or-CFAR Simulations for (N, k, P_{FA}) Combinations

	$N = 8$	$N = 12$	$N = 16$	$N = 20$	$N = 24$
k	4 6 7	7 9 11	9 12 15	11 15 18	16 18 22
10dB	0.0963 0.0450 0.0085	0.2334 0.1111 0.0330	0.2746 0.1543 0.0569	0.3017 0.1813 0.0770	0.3367 0.2015 0.1022
15dB	0.3968 0.3079 0.1467	0.5972 0.4575 0.2844	0.6366 0.5210 0.3591	0.6601 0.5542 0.4062	0.6880 0.5765 0.4524
20dB	0.7281 0.6715 0.5135	0.8441 0.7729 0.6571	0.8627 0.8080 0.7133	0.8733 0.8251 0.7443	0.8855 0.8360 0.7718
25dB	0.9020 0.8792 0.8044	0.9472 0.9208 0.8735	0.9539 0.9341 0.8973	0.9577 0.9404 0.9098	0.9620 0.9445 0.9206
30dB	0.9676 0.9598 0.9328	0.9829 0.9741 0.9579	0.9851 0.9786 0.9662	0.9864 0.9807 0.9705	0.9878 0.9820 0.9741

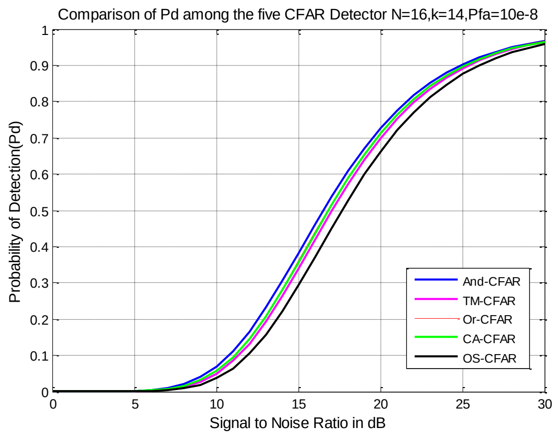

Figure 5: Comparison of five CFAR Detectors

To see the effect of k value on P_D of the four detectors, the k setting is changed to 9 and rest is unchanged. The performance of five detector are shown in Figure 6 . In this And-CFAR is best among five, now CA-CFAR is better than Or-CFAR but Or-CFAR very close in performance with OS-CFAR.

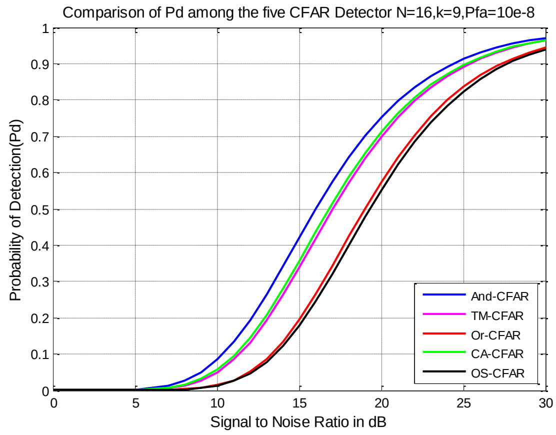

Figure 6: Comparison of five CFAR Detectors

IV. CONCLUSION

In this paper, the properties, performance, and short-comings of the different CFAR schemes for homogeneous and heterogeneous environment are simulated and discussed. The performance of the And-Or CFAR detectors has been examined first and then compared with that of TM, CA and OS-CFAR in homogeneous background. It is proved that the And-Or CFAR detector have a better detection probability compared with the CA, OS and TM-CFAR under different design false alarm probabilities.

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